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Boundary Values of Analytic Functions

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Abstract. It is known that a complex-valued continuous function $S(x)$ as well as a Schwartz distribution on the real axis can be extended in the complex plane minus the support of S to an analytic function $\hat{S}(z)$. In the case of a continuous function the jump of $\hat{S}(z)$ on the real axis represents exactly $S(x)$:

$$\lim_{\varepsilon \to 0+} [\hat{S}(x + i\varepsilon) - \hat{S}(x - i\varepsilon)] = S(x) .$$

We call regular a point x on the support of S such that $\lim_{\varepsilon \to 0+} [\hat{S}(x + i\varepsilon) - \hat{S}(x - i\varepsilon)]$ exists. Conditions are found for the existence of regular points on the support of a distribution. It is possible to call this limit (if this exists) the value $S(x)$ of the distribution S in the point x. Properties of this type occur in the theory of dispersion relations.

§ 1. Introduction

Let $S(x)$ be a complex-valued continuous function with compact support on the real axis R. The function $S(x)$ can be extended in the complex plane [1—3] under the form of a local analytic function $\hat{S}(z)$ throughout the entire complex plane minus the support of S, such that the jump on the real axis is exactly $S(x)$:

$$\lim_{\varepsilon \to 0+} [\hat{S}(x + i\varepsilon) - \hat{S}(x - i\varepsilon)] = S(x) . \tag{1}$$

Relations of the form (1) frequently occur in the study of analytical properties of the scattering amplitude as well as in the theory of dispersion relations [4—5].

The idea to extend functions on the real axis to the complex plane, in such a way that the relation (1) be verified, is an older one. Excellent expositions of this problem can be found in [6—8]. In the case when the function $S(x)$ has a compact support, the analytic continuation $\hat{S}(z)$ can be chosen under integral form

$$\hat{S}(z) = \frac{1}{2\pi i} \int_{-\infty}^{+\infty} \frac{S(t)}{t - z}\, dt, \tag{2}$$