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Correlations in Ising Ferromagnets. III

A Mean-Field Bound for Binary Correlations*

ROBERT B. GRIFFITHS**
Physics Department, Carnegie Institute of Technology, Pittsburgh, Pennsylvania 15213

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Abstract. An inequality relating binary correlation functions for an Ising model with purely ferromagnetic interactions is derived by elementary arguments and used to show that such a ferromagnet cannot exhibit a spontaneous magnetization at temperatures above the mean-field approximation to the Curie or "critical" point. (As a consequence, the corresponding "lattice gas" cannot undergo a first order phase transition in density (condensation) above this temperature.) The mean-field susceptibility in zero magnetic field at high temperatures is shown to be an upper bound for the exact result.

I. Introduction

Many years ago PEIERLS [1] gave a simple argument for the existence of spontaneous magnetization in an Ising ferromagnet at sufficiently low temperatures. More recently this argument has been turned into a rigorous proof [2, 3], and generalized to include interactions other than the nearest-neighbor ferromagnetic coupling originally considered [4].

The existence of a spontaneous magnetization in the "thermodynamic" sense [5] for an Ising ferromagnet implies a horizontal portion of the pressure-density isotherm in the corresponding "lattice gas" [6]. Thus for this somewhat artificial model, the Peierls argument provides an elementary proof that a first order phase transition, or "condensation", takes place at sufficiently low temperatures.

We shall discuss a complementary problem: a proof of the *absence* of spontaneous magnetization (or first-order phase transition for the analogous lattice gas) at a sufficiently *high* temperature. So far as we know, such a proof has not been given previously for any Ising model with interactions of finite range, apart from linear chains [7]. (It is of interest to note that a proof of the absence of spontaneous magnetization for certain systems with a Heisenberg exchange interaction has recently appeared [8].) It is true that for the Ising ferromagnet on a square lattice,

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