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Application of Spectral Representations to the Nonrelativistic and the Relativistic Bethe - Salpeter Equation

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Abstract. The eigenvalue problem of the scalar Bethe-Salpeter equation is solved by application of the vertical Dyson representation. The method of solution is developed in complete analogy to the solution of Schrödinger's equation by a Stieltjes representation in the case of a Yukawa potential. The eigenvalues are zeros of a characteristic determinant, which can be understood as a generalization of the nonrelativistic Jost function.

I. Introduction

Recently Coester [1] has proposed relativistic particle quantum mechanics as a possible alternative of quantum field theory. From the mathematical point of view Coester's approach has the virtue of being based on the firm ground of functional analysis, but physically it suffers from serious shortcomings. It does not offer physical arguments for the choice of interaction operators, nor does it seem possible to include the principle of causality in a simple way. As a consequence of causality matrix elements should satisfy dispersion relations as in field theory.

The opposite situation is encountered in field theory. We consider as an example the formulation of the relativistic two-body system in terms of the Bethe-Salpeter equation. Here the principle of causality is included from the outset and possible approximations for the interaction can be taken from perturbation theory. On the other hand the mathematical structure of the eigenvalue problem is obscure. It is the purpose of this paper to shed some light on this question.

To avoid kinematical and renormalization difficulties we consider the B-S equation for an S-wave bound state in a super-renormalizable theory of three scalar fields with trilinear interaction. Our approach to the solution of the eigenvalue problem is based on a suitable adaption of Jost's method to the relativistic situation. We briefly review the solution of Schrödinger's equation in momentum space for an S-wave bound