Commun. math. Phys. 2, 121—132 (1966)

Irreducible Tensors for the Group SU_3

L. BÁNYAI, N. MARINESCU, I. RASZILLIER, and V. RITTENBERG*

Institute of Physics of the Academy, Bukarest, Rumania

Received October 5, 1965

Abstract. The explicit determination of the matrix elements of the SU_3 irreducible tensors is carried out by a purely algebraic method. These expressions may be used to compute the Clebsch-Gordan coefficients by orthogonalisation. For the special case of $(0, q)$ tensors simple formulas are derived.

I. Introduction

Recently compact Lie groups of rank $\geqq 2$ have found wide applications in elementary particle physics. In view of concrete physical problems, for each group the following main problems have to be solved: (a) determination of the irreducible representations (I.R.) and the matrix elements of the group generators, (b) decomposition of the direct product of two I.R. and hence the computation of the Clebsch-Gordan (C.G.) coefficients. It is well known that the groups of rank $\geqq 2$ are not multiplicity-free (the same representation may occur in the direct product more than once) so that the C.G. coefficients are not completely specified by the basis vectors. The Wigner-Eckart theorem is also modified: the number of reduced matrix elements appearing there is equal to the multiplicity of the equivalent representations.

The simplest of the above groups is SU_3. In this case the problem (a) has already been solved by a number of authors [1, 2, 3, 4, 5], while problem (b) has received until now only an incomplete solution. MOSHINSKY [6] has derived a compact expression for the C.G. coefficients corresponding to the product $(p, q) \otimes (p', 0)$, which is multiplicity-free, while KURIAN, LURIÉ and MACFARLANE [7] have tabulated the coefficients for the simple product $(p, q) \otimes (1, 1)$, BAIRD and BIEDENHARN [8] for the cases $(p, q) \otimes (1, 0)$, $(p, q) \otimes (0, 1)$, $(p, q) \otimes (1, 1)$ and PANDIT and MUKUNDA [9] for the case $(p, q) \otimes (3, 0)$. We must also mention the numerical tables of SU_3 C.G. coefficients [10, 11, 12, 13] for the products of lowest representations. However, the general problem of deriving a simple analytical formula analogous to the Wigner-Racah expression for SU_2 has not yet been solved and it is doubtful if such a task is really possible.

* In partial fulfillment for the requierements of the doctoral degree at the Institute of Physics of the Academy.