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Local Rings and the Connection of Spin with Statistics

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Abstract. The usual investigations of the connection of spin and statistics start with the hypothesis that there exist only fields either commuting or anti-commuting for spacelike separations. Starting from local rings of measurements we want to show that this hypothesis can be added without loss of generality.

I. Introduction

When the concept of quantum fields was introduced in the late twenties, it was an immediate generalization from systems of finite degrees of freedom that one has to introduce commutation relations for fields describing bosons. The problem of how to incorporate Pauli's exclusion principle in such a theory was solved by P. JORDAN and E. WIGNER [1]. The answer was simple and consisted in replacing the commutator by the anti-commutator*. The connection of spin and statistics came, at that stage, as a phenomenological input into the theory.

In the late thirties, however, it was realized that we cannot quantize a given particle, say of spin 1/2, with the rules belonging to Bose statistics if we want at the same time to have positive energy [3]. This meant there was a connection between the three hypotheses,

1) The fields transform in a well defined manner under the inhomogeneous Lorentz transformation.

2) The energy is positive.

3) The fields are quantized with respect either to the rules of Bose statistics or to the rules of Fermi statistics.

A more satisfactory understanding of the connection of the assumptions 1)—3) followed the discovery of the CPT-theorem**.

Postulate 3) says something about the modes of quantization which we admit in our theory. The question arises whether these are the only possible modes or if there exist others. This question has partly been answered by giving examples where one has other commutation relations [5] (see also references in [5]).

* See e.g. [2] for a more detailed account of this part of history.

** The reader will find more information on this subject, with references, in [4].