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Zeta function for the Laplace operator acting on forms in a ball with gauge boundary conditions

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Abstract: The Laplace operator acting on antisymmetric tensor fields in a D-dimensional Euclidean ball is studied. Gauge-invariant local boundary conditions (absolute and relative ones, in the language of Gilkey) are considered. The eigenfuctions of the operator are found explicitly for all values of D. Using in a row a number of basic techniques, as Mellin transforms, deformation and shifting of the complex integration contour and pole compensation, the zeta function of the operator is obtained. From its expression, in particular, $\zeta(0)$ and $\zeta'(0)$ are evaluated exactly. A table is given in the paper for $D = 3, 4, ..., 8$. The functional determinants and Casimir energies are obtained for $D = 3, 4, ..., 6$.

1. Introduction

In this paper we obtain the zeta function of the Laplace operator acting on antisymmetric tensor fields defined in a D-dimensional ball with gauge-invariant boundary conditions. Mathematically this computation is quite an imposing challenge, as is proven by the number of erroneous results reported in the literature on this and related computations (details will be given later). The physical motivations for such a study are to be found in quantum cosmology, where the ζ function of the Laplacian describes the contribution of antisymmetric tensor fields and ghosts to the pre-factor of the wave function of the universe (see e.g. [1]). An intriguing problem in this context is the non-compensation of the boundary contributions to the one–loop divergences between different members of the supergravity supermultiplet [2]. Another motivation is to provide the numerical material needed to extend previous analysis of the heat kernel asymptotics [3] to the case of mixed boundary conditions.

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