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Noncommutative Brownian Motion in Monotone Fock Space

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Dedicated to Professor Minoru Motoo on his 70th birthday

Abstract: An example of noncommutative Brownian motion is constructed on the monotone Fock space which is a kind of "Fock space" generated by all the decreasing finite sequences of positive real numbers. The probability distribution at time $t \geq 0$ associated to this Brownian motion is shown to be the arcsine law normalized to mean 0 and variance t.

1. Introduction

In quantum probability theory or noncommutative probability theory, several (noncommutative) Brownian motions have been known (see Parthasarathy [Par], Meyer [Mey], and Schürmann [Sch2]). For example, the following have been known.

The noncommuting pair of classical Brownian motions arises from the boson Fock space in which the commuting independence and the commuting central limit theorem are concerned (see Segal [Seg], Cushen and Hudson [CuH], Giri and von Waldenfels [GvW], and Hudson and Parthasarathy [HuP]). The associated probability law to this bosonic Brownian motion is the Gaussian distribution.

The noncommuting pair of fermion Brownian motions arises from the fermion Fock space in which the anticommuting independence and the anticommuting central limit theorem are concerned (see Hudson [Hud], von Waldenfels [vW], Barnett, Streater and Wilde [BSW], and Applebaum and Hudson [ApH]). The associated probability law to the fermion Brownian motion is the distribution of Bernoulli type. The noncommuting pair of free Brownian motions arises from the free Fock space in which the free independence and the free central limit theorem are concerned (see Voiculescu [Voi], Speicher [Spe], and Kümmerer and Speicher [KuS]). The associated probability law to the free Brownian motion is the Wigner semicircle law. There is also a one-parameter

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