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# A Diagram Calculus for Certain Canonical Bases

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**Abstract:** We introduce a certain cellular algebra $Q(n, r)$ which is a quotient of the $q$-Schur algebra $S_q(n, r)$. This is naturally equipped with a canonical basis which is compatible with Lusztig's canonical bases for certain modules for the quantized enveloping algebra $U(sl_n)$. We describe a diagram calculus for $Q(n, r)$ which makes calculations involving the corresponding canonical bases easy to understand.

## 1. Introduction

The $q$-Schur algebra $S_q(n, r)$, which was introduced by Dipper and James in [2], is a certain finite-dimensional associative algebra over the ring of Laurent polynomials $\mathcal{A} = \mathbb{Z}[v, v^{-1}]$. Du [3] has described a certain natural free $\mathcal{A}$-basis $\{\theta^w_{\lambda,\mu}\}$ for the $q$-Schur algebra, which we shall refer to as the *canonical basis*. It is known [**8**, Corollary 4.7] that the structure constants with respect to this basis have interpretations in the framework of perverse sheaves and intersection cohomology. It is also known (see for example [**5**, Theorem 4.6]) that the basis $\{\theta^w_{\lambda,\mu}\}$ is very closely related to Lusztig's canonical bases for modules quantized enveloping algebras, which were defined in [**9**]. These bases have turned out to be very important in representation theory.

In Sect.2 we define and study a certain quotient $Q(n, r)$ of $S_q(n, r)$ which inherits a basis from the basis $\{\theta^w_{\lambda,\mu}\}$ of $S_q(n, r)$. This quotient $Q(n, r)$ is closely related to the Temperley–Lieb algebra $TL_r$ of type $A$. Using the main result of [**6**], we show that the multiplication in $Q(n, r)$ can be easily described in terms of a diagram calculus which is an extension of the $r$-diagram calculus for the Temperley–Lieb algebra (described for example in [**12**, Sect. 1]). Furthermore, the diagrams can be interpreted as canonical basis elements of $S_q(n, r)$.

In Sect. 3 we describe the cellular structure of $Q(n, r)$ (in the sense of [7]) and classify the absolutely irreducible modules for $Q(n, r)$ over a field.

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