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Exact Ground State Energy of the Strong-Coupling Polaron*

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Dedicated to the memory of R.L. Dobrushin and P.W. Kasteleyn, two founders of modern statistical mechanics

Abstract: The polaron has been of interest in condensed matter theory and field theory for about half a century, especially the limit of large coupling constant, α. It was not until 1983, however, that a proof of the asymptotic formula for the ground state energy was finally given by using difficult arguments involving the large deviation theory of path integrals. Here we derive the same asymptotic result, $E_0 \sim -C\alpha^2$, and with explicit error bounds, by simple, rigorous methods applied directly to the Hamiltonian. Our method is easily generalizable to other settings, e.g., the excitonic and magnetic polarons.

The polaron Hamiltonian of Fröhlich [1] is a model for the Coulomb interaction of one or more electrons with the quantized phonons of an ionic crystal. In the course of time it was also seen to be an interesting field theory model of non-relativistic particles interacting with a scalar boson field, and it was widely studied [2 – 9] in both contexts.

The model has one dimensionless coupling constant, α, and it was noticed very early that there seems to be a qualitative difference in the ground state between the weak coupling regime, well described by perturbation theory [8], in which the electron is spread out, and the strong coupling regime in which the electron appears to be trapped in a phonon hole of its own making. (This question of trapping seems to have first been considered by Landau [10] in the context of a classical phonon field.)

By now it seems doubtful that such a trapping actually occurs [7, 11, 12], but it remains true that the calculation of the ground state energy, $E_0(\alpha)$, is very different in the two regimes. The strong coupling theory was studied by Pekar [2] (see also [3 – 5, 9]) who hypothesized that in this limit the total ground state wave function Ψ could be taken to be a product of an electronic function $\phi(x)$ and a phonon function $|\xi\rangle$. It is a fact that this ansatz, called the adiabatic approximation, yields exactly the same result for the ground state energy as the corresponding model with classical phonons. This