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Supersymmetry and Localization

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Abstract: We study conditions under which an odd symmetry of the integrand leads to localization of the corresponding integral over a (super)manifold. We also show that in many cases these conditions guarantee exactness of the stationary phase approximation of such integrals.

1. Introduction

Localization formulae express certain integrals over (super)manifolds as sums of contributions of some subsets of these manifolds. Such formulae were studied in various contexts both in mathematics and physics. Some important examples of localization formulae are based on the theory of equivariant cohomology (see [3] for a review of the theory and its applications). One famous particular case is the Duistermaat–Heckman integration formula [5] which became a powerful (though not completely rigorous) tool in Quantum Field Theory (QFT). In the context of QFT, the Duistermaat–Heckman theorem gives sufficient conditions for exactness of semi-classical approximation of field theoretical models (see [4, 11] and references therein for a review of applications of Duistermaat–Heckman and some other localization formulae to QFT).

The aim of the present paper is to derive very general localization formulae in the framework of supergeometry. Namely, we consider an integral over a finite dimensional (super)manifold M, where the integrand is invariant under the action of an odd vector field Q. We formulate sufficient conditions on M and Q under which the integral localizes onto the zero locus of the number part of Q. It is important to stress that without the conditions below, the localization formula can be wrong. (Physicists often used the localization formulae without rigorous justification and without mentioning the conditions of applicability of these formulae).

One of the possible ways to apply the theorems of the present paper is based on the use of the Batalin–Vilkovisky [1] formalism where the calculation of physical

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