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Geometric Quantization of Symplectic Manifolds with Respect to Reducible Non-Negative Polarizations

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Abstract: The leafwise complex of a reducible non-negative polarization with values in the prequantum bundle on a prequantizable symplectic manifold is studied. The cohomology groups of this complex is shown to vanish in rank less than the rank of the real part of the non-negative polarization. The Bohr–Sommerfeld set for a reducible non-negative polarization is defined. A factorization theorem is proved for these reducible non-negative polarizations. For compact symplectic manifolds, it is shown that the above complex has finite dimensional cohomology groups, moreover a Lefschetz fixed point theorem and an index theorem for these non-elliptic complexes is proved. As a corollary of the index theorem, we deduce that the cardinality of the Bohr–Sommerfeld set for any reducible real polarization on a compact symplectic manifold is determined by the volume and the dimension of the manifold.

Contents

1. Introduction

Let M be a symplectic manifold. Assume that M is prequantizable, that is there exists a complex Hermitian line bundle $\mathscr{L}$ over M with a compatible connection ∇, whose curvature is the symplectic form on M. In the program of geometric quantization one is asked to consider a polarization P on M and to study the cohomology groups of the leafwise complex with respect to P with values in $\mathscr{L}$. See e.g. [5, 6, 8]. Traditionally, Kähler polarizations have received most attention, since the

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