Commun. Math. Phys. 183, 365 – 382 (1997)

Communications in
Mathematical
Physics

Chirality and Dirac Operator on Noncommutative Sphere

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Received: 7 May 1996 / Accepted: 17 June 1996

Abstract: We give a derivation of the Dirac operator on the noncommutative 2-sphere within the framework of the bosonic fuzzy sphere and define Connes' triple. It turns out that there are two different types of spectra of the Dirac operator and correspondingly there are two classes of quantized algebras. As a result we obtain a new restriction on the Planck constant in Berezin's quantization. The map to the local frame in noncommutative geometry is also discussed.

1. Introduction

The description of spacetime at the order of the Planck scale and the description of the nature of quantum gravity is a long-standing problem. Quite a number of proposals have been made in order to describe consistently gravitational interaction and quantum field theory. However these proposals either do not give a satisfying formulation of the quantum theory of gravity or, in their present form an interpretation as a theory of the geometry is difficult.

Thus, recently the modification of the concept of geometry itself is also discussed by many authors. Of course one may argue that a successful theory of gravitation will naturally exhibit the necessary structure of a suitable theory of geometry and a natural language to describe it. On the other hand it is not very probable that the standard language of ordinary differential geometry is a suitable tool. The noncommutative geometry from the physicist's point of view is a possibility to describe such a geometry.

The noncommutative geometry is discussed in many contexts. The common idea is that one deals with a function algebra over the space one is interested in and the description of its geometry is made free from the concept of a point [11]. In other words, the geometry of a manifold is reformulated in terms of an algebra of functions defined over it, which may be called structure algebra. Once the algebraic description of the geometry is obtained, the structure algebra can be made noncommutative.

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