Commun. Math. Phys. 183, 253–266 (1997)

Communications in Mathematical Physics

Nonlinear Stability of Solitary Waves of a Generalized Kadomtsev–Petviashvili Equation

Yue Liu[1], Xiao-Ping Wang[2]

[1] Department of Mathematics, University of Texas at Austin Austin, TX 78712, USA

[2] Department of Mathematics, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong

Received: 12 December 1995 / Accepted: 6 June 1996

Abstract: We prove that the set of solitary wave solutions of a generalized Kadomtsev–Petviashvili equation in two dimensions,

$$(u_t + (u^{m+1})_x + u_{xxx})_x = u_{yy}$$

is stable for $0 < m < 4/3$.

1. Introduction

The generalized Kadomtsev–Petviashvili (GKP) equation

$$(u_t + (u^{m+1})_x + u_{xxx})_x = \sigma^2 u_{yy} \tag{1}$$

is a two dimensional analog of the generalized Korteweg–de Vries (GKdV) equation. When $m = 1$ and $\sigma^2 = 1$, (1) is known as the KPI equation while $m = 1$ and $\sigma^2 = -1$ corresponds to the KPII equation. Both are universal models for the propagation of weakly nonlinear dispersive long waves which are essentially one directional, with weak transverse effects [6]. It also describes the evolution of sound waves in antiferromagnetics [9]. It is well known that both KPI and KPII can be solved by the Inverse Scattering Transformation (IST) [1, 2].

Many local existence results for KP and GKP have recently appeared for both infinite space and periodic boundary conditions (see [19, 20, 13]). There are also some global results [22]. It is shown in [9, 20] by a virial method that GKP-I

$$(u_t + (u^{m+1})_x + u_{xxx})_x = u_{yy} \tag{2}$$

has blow-up solutions for $m \geqq 4$ while arguments in [14] indicate that blow up should occur for much lower m, namely $m > \frac{4}{3}$.

An important question is the stability and instability of solitary waves for GKP, that is, solutions of form $u(x, y, t) = \varphi(x - ct, y)$. Existence of solitary waves is shown in [14] for $1 < m < 4$ and also in [21] by a different method. For GKP-I, instability is shown in [14] for $\frac{4}{3} < m < 4$ using a method of Shatah and Strauss [3] and a completed proof is provided by de Bouard and Saut [24]. In this paper, we shall prove that the solitary waves are nonlinearly stable for $0 < m < \frac{4}{3}$. After this