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A Maxwellian Lower Bound for Solutions to the Boltzmann Equation

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Abstract: We prove that the solution of the spatially homogeneous Boltzmann equation is bounded pointwise from below by a Maxwellian, i.e. a function of the form $c_1 \exp(-c_2 v^2)$. This holds for any initial data with bounded mass, energy and entropy, and for any positive time $t \geqq t_0$. The constants, c_1, and c_2, depend on the mass, energy and entropy of the initial data, and on $t_0 > 0$ only.

A similar result is obtained for the Kac caricature of the Boltzmann equation, where the proof is easier.

1. Introduction

We consider the spatially homogeneous Boltzmann equation,

$$\partial_t f = Q(f,f)\,, \tag{1.1}$$

where $f = f(v,t)$, $v \in \mathbb{R}^3$, is a non-negative function which gives the velocity distribution of a (spatially homogeneous) dilute gas. The bilinear operator Q is the so-called collision operator. It is given by

$$Q(f,g)(v) = \iint (f(v')g(v_1') - f(v)g(v_1))B(|v-v_1|,\theta)\,d\omega\,dv_1\,, \tag{1.2}$$

where v' and v_1' are the velocities after the collision of two particles which had the velocities v and v_1 before the collision. The velocities before and after a collision are related by

$$v' = v + [(v-v_1)\cdot\omega]\omega\,,$$

$$v_1' = v_1 - [(v-v_1)\cdot\omega]\omega\,.$$

The collision operator Q has the form (1.2) for all monatomic gases. The exact form of the interaction between the particles is given by the collision kernel, B. In this paper we deal only with the so-called hard potentials with an angular cut-off. In this case,

$$B(|v-v_1|,\theta) = h(\theta)|v-v_1|^\beta\,, \tag{1.3}$$