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The Trivial Connection Contribution to Witten's Invariant and Finite Type Invariants of Rational Homology Spheres

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Abstract: We derive an analog of the Melvin–Morton bound on the power series expansion of the colored Jones polynomial of algebraically split links and boundary links. This allows us to produce a simple formula for the trivial connection contribution to Witten's invariant of rational homology spheres. We show that the n^{th} term in the $1/K$ expansion of the logarithm of this contribution is a finite type invariant of Ohtsuki order $3n$ and of at most Garoufalidis order n.

1. Introduction

Let M be a 3-dimensional manifold with an N-component link $\mathscr{L}$ inside it. We assign α_j-dimensional irreducible representations of $SU(2)$ to every component $\mathscr{L}_j$ of $\mathscr{L}$. Witten's invariant of M and $\mathscr{L}$ is given [1] by a path integral over all $SU(2)$ connections A_μ on M:

$$Z_{\alpha_1,\ldots,\alpha_N}(M,\mathscr{L};k) = \int [\mathscr{D}A_\mu] \exp\left(\frac{ik}{2\pi} S_{\text{CS}}\right) \prod_{j=1}^{N} \operatorname{Tr}_{\alpha_j} \operatorname{Pexp}\left(\oint_{\mathscr{L}_j} A_\mu dx^\mu\right) , \quad (1.1)$$

here S_{CS} is the Chern–Simons action

$$S_{\text{CS}} = \frac{1}{2} \operatorname{Tr} \varepsilon^{\mu\nu\rho} \int_M d^3x \left(A_\mu \partial_\nu A_\rho + \frac{2}{3} A_\mu A_\nu A_\rho\right) , \quad (1.2)$$

$\operatorname{Tr}_{\alpha_j} \operatorname{Pexp}\left(\oint_{\mathscr{L}_j} A_\mu dx^\mu\right)$ are traces of holonomies of A_μ along $\mathscr{L}_j$ taken in α_j-dimensional representations of $SU(2)$ and Tr of Eq. (1.2) is the trace taken in the fundamental 2-dimensional representation. In most cases instead of the integer number k we will be using

$$K = k + 2 . \quad (1.3)$$

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