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Necessary Conditions for Existence of Non-Degenerate Hamiltonian Structures

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Abstract: The necessary criteria are pointed out for the existence of Hamiltonian and bi-Hamiltonian non-degenerate structures for a nonlinear system of partial differential equations of first order. The results are formulated in terms of the new invariants of the intrinsic geometry, introduced in this paper, connected with the Nijenhuis and Haantjes tensors of a (1,1) tensor field.

1. Introduction

This paper is devoted to the investigation of the intrinsic geometry of systems of nonlinear partial differential equations of first order

$$u_t^i = \sum_{j=1}^{n} A_j^i(u^1, \dots, u^n) u_y^j \, . \tag{1.1}$$

As it is known, systems (1.1) arise in numerous classical problems of gas dynamics and mathematical physics [1–4, 7–9, 12–15].

Riemann pointed out in his classical work [1] that the system (1.1) is closely connected with the (1,1) tensor field $A_j^i(u^1, \dots, u^n)$ defined on the Euclidean space R^n with the coordinates $u^1, \dots, u^n$.

Geometry of the vector fields of eigenvectors of the operators $A_j^i(u^1, \dots, u^n)$ has been studied in famous papers by Nijenhuis [5] and Haantjes [6].

Hamiltonian systems (1.1) and the associated structures of the Poisson brackets were investigated in [7–9, 14, 15] along with their applications to the theory of the Whitham equations.

Tensor fields $A_j^i(u^1, \dots, u^n)$ were considered in [5, 6] as vector-valued differential 1-forms and also as fields of operators defined on the tangent bundle $T(M^n)$. The Nijenhuis tensor $N_{jk}^i(u^1, \dots, u^n)$ and the Haantjes tensor $H_{jk}^i(u^1, \dots, u^n)$ were considered as the vector-valued differential 2-forms.

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