

On Isospectral Sets of Jacobi Operators

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Abstract: We consider the inverse spectral problem for a class of reflectionless bounded Jacobi operators with empty singularly continuous spectra. Our spectral hypotheses admit countably many accumulation points in the set of eigenvalues as well as in the set of boundary points of intervals of absolutely continuous spectrum. The corresponding isospectral set of Jacobi operators is explicitly determined in terms of Dirichlet-type data.

1. Introduction

The principal aim of this paper is to study certain bounded self-adjoint Jacobi operators whose inverse spectral theory and isospectral class can be characterized explicitly.

Since the literature on inverse spectral theory for Jacobi operators (especially in the periodic and short-range scattering case) is rather extensive, we confine ourselves to a brief account of those results which are close in spirit to our approach. In this context, the use of auxiliary spectral problems of the Dirichlet-type in connection with either the moment problem or the algebro-geometric approach to (quasi-)periodic finite-gap Jacobi operators, comes to mind first. Dirichlet spectra and the moment problem were first combined in the pioneering work by Kac and van Moerbeke [23, 24, 29]. The Jacobi inversion problem in connection with Dirichlet divisors appeared in Date and Tanaka [12] (see also [36]) and simultaneously in Dubrovin, Matveev, and Novikov [15] with further developments in [28, 30, 31]. (The algebro-geometric method is presented in great detail in [6].) A complete algebro-geometric treatment of Toda and Kac–van Moerbeke hierarchies can be found in [8]; the isospectral torus of quasi-periodic Jacobi operators is explicitly realized in [20]. The next step involved extensions to certain almost

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