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Link Invariants and Combinatorial Quantization of Hamiltonian Chern Simons Theory

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Abstract: We define and study the properties of observables associated to any link in $\Sigma \times \mathbf{R}$ (where Σ is a compact surface) using the combinatorial quantization of hamiltonian Chern–Simons theory. These observables are traces of holonomies in a non-commutative Yang–Mills theory where the gauge symmetry is ensured by a quantum group. We show that these observables are link invariants taking values in a non-commutative algebra, the so-called Moduli Algebra. When $\Sigma = S^2$ these link invariants are pure numbers and are equal to Reshetikhin–Turaev link invariants.

1. Introduction

Since the fundamental discovery by V. Jones in 1984 of a new link invariant, there has been a tremendous interest and activity in low dimension topology using field theory techniques. The original definition of the Jones Polynomial was purely combinatorial and a geometrical understanding of it was finally given by E. Witten in 1989 [22]. He showed that the Jones polynomial could be interpreted as the correlation function of Wilson loops (i.e. traces of holonomies) in Chern–Simons theory. His work opened a new area of research in what is now called three dimensional topological field theory. Although this theory is purely topological (i.e. in a hamiltonian picture the hamiltonian is zero) and therefore contains no dynamics, the quantization of this theory is not at all a trivial task, mainly because there is no direct procedure to quantify this theory. The original method of E. Witten is a brilliant use of path integrals, heuristic regularization (by a framing) of Wilson loops and relations with conformal field theory. Although very appealing and having far reaching consequences, his formalism is not at all mathematically well defined and this is one of the reasons why many researchers in this field have used other approaches. These methods can be roughly divided in two classes: perturbative and non-perturbative methods. On the one hand perturbative methods

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