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The Higher Order Hamiltonian Structures for the Modified Classical Yang–Baxter Equation

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Abstract: We consider constructing the higher order Hamiltonian structures on the dual of the Lie algebra from the first Hamiltonian structure of the coadjoint orbit method. For this purpose we show that the structure of the Lie algebra g is inherited to the algebra of vector fields on g^* through the solution of the Modified Classical Yang–Baxter equation (Classical r matrix). We study the algebra that generates the compatible Poisson brackets.

Introduction

Let D be a ring of differential operators and E be a ring of pseudo-differential operators. We have a direct sum decomposition such as

$$E = D \oplus E_{-1},$$

where E_{-1} is a subring of E consisted of pseudo-differential operators whose orders are at most -1. For $P \in E$, we abbreviate $Proje_D P$ and $Proje_{E_{-1}} P$ as P_+ and P_- respectively. Let L be a monic p^{th} order differential operator, $L = \partial^p + a_{p-1}(x)\partial^{p-1} + \cdots + a_0(x)$, where $\partial = \frac{\partial}{\partial x}$. We define the space of δ functions K such as

$$K = \left\{ \sum_{i_1, \ldots, i_m} a_{i_1 \ldots i_m} \delta^{(i_1)}(x_{i_1}) \cdots \delta^{(i_m)}(x_{i_m}) | a_{i_1 \ldots i_m} \in \mathbf{C} \right\}.$$

We regard that

$$K = \bigoplus_{n \geqq 0} \otimes^n C^{-\infty}(\mathbf{R}),$$

where $C^{-\infty}(\mathbf{R})$ is distribution of $\mathbf{R}$. Let M be a space of functional of L such as

$$M = \left\{ F(L) = \sum_{i_1, \ldots, i_m, j_1, \ldots, j_m} f_{i_1, \ldots, i_m}^{j_1, \ldots, j_m} a_{i_1}^{(j_1)}(x_{i_1}) \cdots a_{i_m}^{(j_m)}(x_{i_m}) | f_{i_1, \ldots, i_m}^{j_1, \ldots, j_m} \in K \right\}.$$

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