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Limits on Stability of Positive Molecular Ions in a Homogeneous Magnetic Field

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Abstract: The problem of stability of positive diatomic molecular ions with the nuclear charges Z_1 and Z_2 and N electrons in a homogeneous magnetic field B is studied for $Z_1, Z_2, N, B \to \infty$. The conditions of instability are obtained for different relations among Z_1, Z_2, N and B. A new version of the HVZ theorem for systems in a homogeneous magnetic field is proved.

1. Introduction

It is well known [5] that a positive diatomic molecular ion with N electrons and nuclear charges Z_1 and Z_2 is unstable for large Z_1 and Z_2. It means that for every positive N there exists a constant $Z^c > 0$ such that whenever both $Z_1 > Z^c$ and $Z_2 > Z^c$, the hamiltonian of the system (after the separation of the center of mass motion) has no discrete spectrum. Furthermore

$$\overline{\lim_{N\to\infty}}\ Z^c N^{-1} \leqq 2\,. \tag{1.1}$$

In this paper we discuss a generalization of this result to molecules in a magnetic field. This case is more complicated for the following reasons. First we cannot use the standard definition of stability, because the center of mass motion cannot be separated in a homogeneous magnetic field in a standard sense. A key to this problem is a new version of the *HVZ* theorem for the hamiltonian on the subspace of functions with fixed rotational $SO(2)$ symmetry which is proved in Sect. A1 (see Appendix). The idea of fixing of the type of the $SO(2)$ symmetry as a substitute for the separation of the center of mass motion for the *HVZ* theorem was suggested earlier by G. Zhislin and the author [6]. But in [6] the *HVZ* theorem was proved only for the case when either all the charges of the particles have the same sign or the magnetic field B increases at infinity.

In this paper we consider molecular ions consisting of positive nuclei and negative electrons in a homogeneous magnetic field and the results of [6] can not be applied.

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