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The Microlocal Spectrum Condition and Wick Polynomials of Free Fields on Curved Spacetimes

R. Brunetti[1], **K. Fredenhagen**[2], **M. Köhler**[2]

[1] Dip di Scienze Fisiche, Università di Napoli "Federico II", Pad 19, Mostra d'Oltremare, I-80125 Napoli, Italy. E-mail: brunetti@axpna1.na.infn.it
[2] IL Institut für Theoretische Physik, Universität Hamburg, Lubuper Chaussee 149, D-22761 Hamburg, Germany. E-mail: i02fre@dsyibm.desy de, mkoehler@x4u.desy.de

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Abstract: Quantum fields propagating on a curved spacetime are investigated in terms of microlocal analysis. We discuss a condition on the wave front set for the corresponding n-point distributions, called "microlocal spectrum condition" (μSC). On Minkowski space, this condition is satisfied as a consequence of the usual spectrum condition. Based on Radzikowski's determination of the wave front set of the two-point function of a free scalar field, satisfying the Hadamard condition in the Kay and Wald sense, we construct in the second part of this paper all Wick polynomials including the energy-momentum tensor for this field as operator valued distributions on the manifold and prove that they satisfy our "microlocal spectrum condition."

1. Introduction

Quantum Field Theory on curved spacetime (QFT on CST) describes quantum fields propagating under the influence of an external gravitational field. The main problem which has to be resolved in this setting is an appropriate formulation of stability. On Minkowski space, stability is expressed by the requirement of positive energy, i.e., the generators of time-like translations are represented as positive operators on some distinguished Hilbert space. In the absence of a time-like Killing vector field a corresponding condition can not be formulated and there exists no preferred Hilbert space. These difficulties can best be treated in the algebraic approach to quantum field theory [HK64]. In this approach, the formulation of a particular model can be divided into two steps. In the first step an algebra of observables is constructed in terms of commutation or anticommutation relations. This step was performed by Dimock [Dim80, Dim82, Dim92] for the free scalar, Dirac and electro-magnetic fields on globally hyperbolic spacetimes. In a second step, a class of states with suitable stability properties has to be found. States are here considered as expectation functionals, i.e., normalized positive linear functionals on the algebra of observables. Via the GNS construction each state induces a representation of the observable algebra