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Localization of Classical Waves I: Acoustic Waves

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Abstract: We consider classical acoustic waves in a medium described by a position dependent mass density $\varrho(x)$. We assume that $\varrho(x)$ is a random perturbation of a periodic function $\varrho_0(x)$ and that the periodic acoustic operator $A_0 = -\nabla \cdot \frac{1}{\varrho_0(x)}\nabla$ has a gap in the spectrum We prove the existence of localized waves, i e., finite energy solutions of the acoustic equations with the property that almost all of the wave's energy remains in a fixed bounded region of space at all times, with probability one Localization of acoustic waves is a consequence of Anderson localization for the self-adjoint operators $A = -\nabla \cdot \frac{1}{\varrho(x)}\nabla$ on $L^2(\mathbb{R}^d)$ We prove that, in the random medium described by $\varrho(x)$, the random operator A exhibits Anderson localization inside the gap in the spectrum of A_0 This is shown even in situations when the gap is totally filled by the spectrum of the random operator, we can prescribe random environments that ensure localization in almost the whole gap.

Contents

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