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On Finite 4D Quantum Field Theory in Non-Commutative Geometry

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Abstract: The truncated 4-dimensional sphere S^4 and the action of the self-interacting scalar field on it are constructed The path integral quantization is performed while simultaneously keeping the $SO(5)$ symmetry and the finite number of degrees of freedom The usual field theory UV-divergences are manifestly absent

1. Introduction

The basic ideas of non-commutative geometry were developed in [1, 2], and in the form of the matrix geometry in [3, 4] The applications to physical models were presented in [2, 5], where the non-commutativity was in some sense minimal the Minkowski space was not extended by some standard Kaluza–Klein manifold describing internal degrees of freedom, but just by two discrete points. The algebra of functions on this manifold remains commutative, but the complex of the differential forms does not This led to a new insight on the $SU(2)_L \otimes U(1)_R$ symmetry of the standard model of electroweak interactions The consideration of gravity was included in [6] Such models, of course, do not lead to UV-regularization, since they do not introduce any modification of the space-time short-distance behaviour

To achieve the UV-regularization one should introduce a non-commutative deformation of the algebra of functions on a space-time manifold in the Minkowski case, or on the space manifold in the Euclidean version One of the simplest locally Euclidean manifolds is the sphere S^2 Its non-commutative (fuzzy) deformation was described by [7, 8] in the framework of the matrix geometry. A more general construction of some non-commutative homogeneous spaces was described in [9] using coherent-states technique

The first attempts to construct fields on a truncated sphere were presented in [8,10] within the matrix formulation Using a more general approach, the fields on truncated S^2 were investigated in detail in [11–13] In particular, in [11] it was the

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