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Dynamic Stability of Vortex Solutions of Ginzburg–Landau and Nonlinear Schrödinger Equations

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Abstract: The dynamic stability of vortex solutions to the Ginzburg–Landau and nonlinear Schrödinger equations is the basic assumption of the asymptotic particle plus field description of interacting vortices. For the Ginzburg–Landau dynamics we prove that all vortices are asymptotically nonlinearly stable relative to small radial perturbations Initially finite energy perturbations of vortices decay to zero in $L^p(\mathbb{R}^2)$ spaces with an algebraic rate as time tends to infinity We also prove that under general (nonradial) perturbations, the plus and minus one-vortices are linearly dynamically stable in L^2, the linearized operator has spectrum equal to $(-\infty, 0]$ and generates a C_0 semigroup of contractions on $L^2(\mathbb{R}^2)$ The nature of the zero energy point is clarified, it is *resonance*, a property related to the infinite energy of planar vortices Our results on the linearized operator are also used to show that the plus and minus one-vortices for the Schrödinger (Hamiltonian) dynamics are spectrally stable, i e the linearized operator about these vortices has (L^2) spectrum equal to the imaginary axis The key ingredients of our analysis are the Nash–Aronson estimates for obtaining Gaussian upper bounds for fundamental solutions of parabolic operators, and a combination of variational and maximum principles

1. Introduction

In this paper, we study the dynamic stability of vortex solutions of the Ginzburg–Landau and nonlinear Schrödinger equations

$$u_t = \Delta u + (1 - |u|^2)u = \frac{\delta \mathscr{E}}{\delta \bar{u}}\,, \tag{1 1}$$

$$-iu_t = \Delta u + (1 - |u|^2)u = \frac{\delta \mathscr{E}}{\delta \bar{u}} \tag{1 2}$$

Here, $u = u(t,x)$ is a complex valued function defined for each $t > 0$ and $x = (x_1, x_2) \in \mathbb{R}^2$ $\Delta = \partial_{x_1}^2 + \partial_{x_2}^2$ denotes the two-dimensional Laplacian The energy