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Tetrahedral and Cubic Monopoles

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Abstract: Using a numerical implementation of the ADHMN construction, we compute the fields and energy densities of a charge three monopole with tetrahedral symmetry and a charge four monopole with octahedral symmetry. We then construct a one parameter family of spectral curves and Nahm data which represent charge four monopoles with tetrahedral symmetry, which includes the monopole with octahedral symmetry as a special case. In the moduli space approximation, this family describes a novel kind of four monopole scattering and we use our numerical scheme to construct the energy density at various times during the motion

1. Introduction

BPS monopoles are topological solitons in a Yang–Mills–Higgs gauge theory in three space dimensions The equation for static monopoles is integrable, so that a variety of techniques are available for studying monopoles and constructing solutions Monopoles of charge one and two are well-understood, with explicit solutions known, but for higher charges the situation is not so clear Despite the integrability of the equation, explicit solutions for charge three and above are known only in the axisymmetric case, which corresponds to coincident monopoles Very recently, some progress has been made in this area [4] with existence proofs for a charge three monopole with tetrahedral symmetry and a charge four monopole with octahedral symmetry In this paper, we compute these monopoles using a numerical implementation of the Atiyah–Drinfeld–Hitchin–Manin–Nahm (ADHMN) construction and display their energy densities.

When time dependence is introduced, the monopole equation of motion is not integrable However, analytical progress can still be made, via the moduli space approximation [7, 9], from knowledge of the static monopoles. This has been extensively studied for the case of charge two monopole scattering [1], but the extension to higher charges has proved a less tractable problem We have made

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