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Witten's Invariants of Rational Homology Spheres at Prime Values of K and Trivial Connection Contribution

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Abstract: We establish a relation between the coefficients of asymptotic expansion of the trivial connection contribution to Witten's invariant of rational homology spheres and the invariants that T Ohtsuki extracted from Witten's invariant at prime values of K We also rederive the properties of prime K invariants discovered by H Murakami and T Ohtsuki We do this by using the bounds on Taylor series expansion of the Jones polynomial of algebraically split links, studied in our previous paper These bounds are enough to prove that Ohtsuki's invariants are of finite type The relation between Ohtsuki's invariants and trivial connection contribution is verified explicitly for lens spaces and Seifert manifolds

1. Introduction

Witten's invariant of 3d manifolds defined in [1] by a path integral over the $SU(2)$ connections A_μ on a 3d manifold M

$$Z(M,k) = \int [\mathcal{D}A_\mu] e^{\frac{ik}{2\pi} S_{CS}[A_\mu]} , \tag{1 1}$$

$$S_{CS} = \frac{1}{2} \operatorname{Tr} \varepsilon^{\mu\nu\rho} \int_M d^3x \left(A_\mu \partial_\nu A_\rho + \frac{2}{3} A_\mu A_\nu A_\rho \right) \tag{1 2}$$

($k \in \mathbb{Z}$, Tr is the trace taken in the fundamental representation of $SU(2)$) can also be calculated combinatorially with the help of the surgery formula. Let M be a 3d manifold constructed by $(p_j, 1)$ surgeries on the components $\mathcal{L}_j$ of an N-component link $\mathcal{L}$ in S^3. A $(p, 1)$ surgery means that the meridian of the tubular neighborhood is glued to the parallel plus p meridians on the boundary of the knot complement (in other words, a Dehn's surgery is performed on a knot with framing number p) The invariant of M reduced to canonical 2-framing can be expressed in terms of

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