Commun. Math. Phys. 179, 725–735 (1996)

Communications in
Mathematical
Physics

Lower Bounds on the Width of Stark–Wannier Type Resonances

J. Asch, P. Briet

Centre de Physique Théorique, CNRS - Luminy, Case 907, F-13288 Marseille Cedex 9, France.
E-mail: asch@cpt.univ-mrs.fr

Received: 1 November 1995/Accepted: 29 January 1996

Abstract: We prove that the Schrödinger operator $-d^2/dx^2 + Fx + W(x)$ on $L^2(\mathbf{R})$ with W bounded and analytic in a strip has no resonances in a region $\operatorname{Im} E \geq -\exp(-C/F)$.

1. Introduction and Main Result

The resonance problem of a Schrödinger particle subject to an electric field with non-vanishing mean bears interesting physical and mathematical aspects and has attracted much activity in both fields. From the point of view of transport in solids not only fluctuations of short range or Coulomb type but also random, quasiperiodic and especially periodic potentials, the Wannier case [25], are of interest. For the physics we refer to Avron [3], Grecchi and Sacchetti [15] and their references. Mathematically the classical questions are definition, existence and location of resonances. They are non–trivial even in a one dimensional situation.

For the case of fluctuations which do not decay at infinity the definition setup for resonances by spectral deformation was essentially given by Herbst and Howland [18].

The existence of resonances in the Wannier case was discussed by Avron [3]. Rigorous results were obtained in the high field regime by Agler, Froese [1]; in a small field and semiclassical context by Combes and Hislop [10], Bentosela and Grecchi [5]; for potentials with a finite number of bands by Buslaev and Dmitrieva [9], Grecchi and Maioli and Sacchetti [14] who have also results for the disordered case with large periods [13]. The techniques in [10, 5, 13, 14] were spectral deformation and perturbation theory; in [9] the complex poles of the reflection coefficient were studied directly by ODE methods; in [1] a Birman–Schwinger technique was employed.

Concerning the location in the Wannier case it is suggested by the Zener tilted band picture that the width –or imaginary part, or the inverse lifetime of the resonances– is exponentially small in the strength of the homogeneous part of the field [3]. The works on the existence confirm this: upper bounds on the imaginary part were given in [10, 5, 14] for the semiclassical and for the finite band case.

Correspondence to: asch@cpt.univ-mrs.fr