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Global Weak Solutions and Attractors of the Three Dimensional Maxwell–Bloch Two Level Laser Systems

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Abstract: The three-dimensional Maxwell–Bloch system governs the multi-longitudinal and transverse mode dynamics of two level wide aperture lasers in an optical ring cavity. The system is hyperbolic in the propagation direction, and dispersive in the transverse directions due to diffraction effects. A rich variety of optical patterns and chaos are present in the dynamics. We show the global existence of weak solutions in L^p $(2 \leqq p < \infty)$ spaces of the Maxwell–Bloch system under both absorbing and periodic boundary conditions. The weak solutions are unique within the class of solutions provided by our regularization procedure and approach a universal attractor which has only partial smoothing instead of the C^∞ smoothing property found in early works for the (longitudinal) one-dimensional and (transverse) two-dimensional cases. The idea of the proof makes essential use of both the hyperbolicity and dispersivity of the system. In the case of periodic boundary condition, our result depends on a conjectural Strichartz inequality.

1. Introduction

In this paper, we are concerned with the dynamics of the three dimensional Maxwell–Bloch(MB) two level laser system:

$$\mathscr{E}_t + \mathscr{E}_z - ia\Delta_\perp \mathscr{E} = -\sigma\mathscr{E} + \sigma\mathscr{P}\,, \tag{1.1}$$

$$\mathscr{P}_t + (1 + i\Omega)\mathscr{P} = (r - \mathscr{N})\mathscr{E}\,, \tag{1.2}$$

$$\mathscr{N}_t + b\mathscr{N} = \tfrac{1}{2}(\mathscr{E}^\star\mathscr{P} + \mathscr{E}\mathscr{P}^\star)\,, \tag{1.3}$$

where $\Delta_\perp =$ the two dimensional Laplacian in $(x, y) \in R^2$ or T^2 (T^n will denote the unit n-dimensional torus), (x, y) being the transverse dimensions; the real parameter a measures the transverse diffraction; $z \in T^1$, the longitudinal or propagational dimension that has been normalized to size one. The complex variables $\mathscr{E}$ and $\mathscr{P}$ are the electric and polarization fields, and $\mathscr{N}$ is proportional to the difference between the atomic and the initial inversion; the positive parameters σ and b are respectively the dissipation(decay) rates of the electric field and population inversion both