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Mean of the Singularities of a Gibbs Measure

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Abstract: We calculate the value of the average of the singularities of a Gibbs measure μ invariant with respect to an expansive C^2 diffeomorphism of a one-compact manifold. This is the value related to dimension that one computes numerically. We then define and study a function, known as the correlation dimension, which is related to a free energy function, and we generalize the results in higher dimension with an axiom A transformation acting on a two-compact manifold.

0. Introduction

Let μ be a measure on a compact space X. Multifractal analysis is concerned with the description of different decay rates of the measures $\mu(B(x,r))$ of balls of radius r as r goes to 0. A natural quantity to be considered is

$$M(r,\beta) = \frac{\operatorname{Log} \int \mu(B(x,r))^{\beta}\, \mu(dx)}{\operatorname{Log} r}\,.$$

It can be argued [P, G] that in numerical computations based on time-series associated to a dynamical system, the functions $M(r,\beta)$ are the most accessible.

We prove here the existence of the limit

$$\forall \beta \in \mathbb{R}, \quad M(\beta) = \lim_{r \to 0} M(r,\beta)\,, \tag{0.1}$$

and we compute $M(\beta)$ in terms of other dynamical quantities. Actually, it is known in [P] that this function M referred to as the correlation dimension, plays an important role in the numerical investigation of some models, and differs in general with other characteristic dimensions, as a Hausdorff dimension, capacity or information dimension. There exists also a numerical procedure in [G] and described in [P] which is simple and runs fast.

The aim of this paper is to compute this correlation dimension in the case when the measure μ is a Gibbs measure for an expansive smooth transformation in dimension 1, or a two dimensional hyperbolic diffeomorphism. The method used