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Quantum $\mathcal{W}_N$ Algebras and Macdonald Polynomials

H. Awata[1,★], **H. Kubo**[2,★], **S. Odake**[3], **J. Shiraishi**[4]

[1] Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606, Japan
E-mail: awata@yukawa.kyoto-u ac.jp
[2] Department of Physics, Faculty of Science, University of Tokyo, Tokyo 113, Japan
E-mail: kubo@danjuro phys s.u-tokyo.ac.jp
[3] Department of Physics, Faculty of Science, Shinshu University, Matsumoto 390, Japan
E-mail: odake@yukawa.kyoto-u ac.jp
[4] Institute for Solid State Physics, University of Tokyo, Tokyo 106, Japan
E-mail: shiraish@momo.issp.u-tokyo.ac.jp

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Abstract: We derive a quantum deformation of the $\mathcal{W}_N$ algebra and its quantum Miura transformation, whose singular vectors realize the Macdonald polynomials.

1. Introduction

The excited states of the Calogero–Sutherland model [14] and its relativistic model (the trigonometric limit of the Ruijsenaars model) [11] are described by the Jack polynomials [13] and their q-analog (the Macdonald polynomials) [6], respectively. Since the Jack polynomials coincide with certain correlation functions of the $\mathcal{W}_N$ algebra [8, 1], it is natural to expect that the Macdonald polynomials are also realized by those of a deformation of $\mathcal{W}_N$ algebra.

In a previous paper [12], we derived a quantum Virasoro algebra whose singular vectors are some special kinds of Macdonald polynomials. On the other hand, E. Frenkel and N. Reshetikhin succeeded in constructing the Poisson $\mathcal{W}_N$ algebra and its quantum Miura transformation in the analysis of the $U_q(\widehat{sl}_N)$ algebra at the critical level [4]. Like the classical case [3], these two works, q-Virasoro and q-Miura transformation, are essential to find and study a quantum $\mathcal{W}_N$ algebra. In this article, we present a q-$\mathcal{W}_N$ algebra[1] whose singular vectors realize the general Macdonald polynomials.

This paper is arranged as follows: In Sect. 2, we define a quantum deformation of $\mathcal{W}_N$ algebras and its quantum Miura transformation. The screening currents and a vertex operator are derived in Sects. 3 and 4. A relation with the Macdonald polynomials is obtained in Sect. 5. Section 6 is devoted to conclusion and discussion. Finally we recapitulate the q-Virasoro algebra and the integral formula for the Macdonald polynomials in the appendices.

★ JSPS fellow.

[1] After finishing of this work, we received the preprint "*Quantum $\mathcal{W}$-algebras and elliptic algebras*" by B Feigin and E Frenkel (q-alg/9508009) They discuss similar things as Sects 2 1, 2 3, 3 1 and Eq (8) of ours Although the algebra of screening currents is considered there, the normal ordering of q-$\mathcal{W}$ generators and the relation with the Macdonald polynomial are not given