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Absolutely Continuous Spectrum of One-Dimensional Schrödinger Operators and Jacobi Matrices with Slowly Decreasing Potentials

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Abstract: We prove that for any one-dimensional Schrödinger operator with potential $V(x)$ satisfying decay condition $|V(x)| \leqq Cx^{-3/4-\varepsilon}$, the absolutely continuous spectrum fills the whole positive semi-axis. The description of the set in $\mathbb{R}^+$ on which the singular part of the spectral measure might be supported is also given. Analogous results hold for Jacobi matrices.

Introduction

Let $H_V = -\frac{d^2}{dx^2} + V(x)$ be the one-dimensional Schrödinger operator acting on $L^2(0,\infty)$. We assume $V(x)$ is a real-valued locally integrable function which goes to zero at infinity. It is a well-known fact that if we fix some self-adjoint boundary condition at zero, the expression H_V has unique self-adjoint realization in $L^2(0,\infty)$. The essential spectrum of the operator H_V, $\sigma_{\text{ess}}(H_V)$, coincides with the positive semi-axis since the potential vanishing at infinity constitutes a relatively compact perturbation of the free Hamiltonian.

In this paper, we explore the problem of dependence of the spectral properties of H_V for positive energies on the rate of decay of the potential V. In particular, the interesting question is to determine the critical rate of decay which can lead to the complete or partial destruction of the absolutely continuous spectrum on the positive half-axis, and, correspondingly, to find out which classes of potentials are not strong enough to seriously affect the absolutely continuous spectrum inherent for the free Hamiltonian. As is generally known, if $V(x)$ belongs to $L^1(0,\infty)$ then the spectrum on the positive semi-axis is purely absolutely continuous (see, e.g., [29]). The situation is not so clear for decreasing potentials which are not absolutely integrable. There are many results on the absolute continuity of the spectrum on the positive semi-axis (except perhaps for a finite number of resonances in some cases) for certain classes of decaying potentials, such as potentials of bounded variation [29] or specific oscillating potentials (see, e.g., [1, 11, 30, 16] for further references). But no general relations between the rate of decay and spectral properties, apart from the absolutely integrable class, seem to be known.