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Convergence of Shock Capturing Schemes for the Compressible Euler–Poisson Equations

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Abstract: We are concerned with approximate methods to construct global solutions with geometrical structure to the compressible Euler–Poisson equations in several space variables. A shock capturing numerical scheme is introduced to overcome the new difficulties from the nonlinear resonance of the system and the nonlocal behavior of the source terms. The convergence and consistency of the shock capturing scheme for the equations is proved with the aid of the compensated compactness method. Then new existence results of the global solutions with geometrical structure are obtained. The traces of the weak solutions are defined and then the weak solutions are proved to satisfy the boundary conditions. The initial data are arbitrarily large with L^∞ bounds.

1. Introduction

We consider the Euler–Poisson equations for compressible flows with the form

$$\begin{cases} \rho_t + \nabla \cdot \vec{m} = 0 \,, \\ \vec{m}_t + \nabla \cdot (\frac{\vec{m}\otimes\vec{m}}{\rho}) + \nabla p(\rho) = \rho \nabla \phi - \frac{\vec{m}}{\tau} \,, \\ \Delta\phi = \rho - D(\vec{x}) \,, \end{cases} \tag{1.1}$$

where $\rho(\vec{x},t) \in \mathbf{R}$, $\vec{m}(\vec{x},t) \in \mathbf{R}^N$, and $\phi(\vec{x},t) \in \mathbf{R}$ denote the density, the mass, and the potential of the flows, respectively; $p(\rho) = \rho^\gamma/\gamma$, $\gamma \in (1, \frac{5}{3}]$, is the pressure-density relation function; $\tau > 0$ is the momentum relaxation time; $D(\vec{x})$ is the doping profile; and $\Delta\phi = \frac{\partial^2\phi}{\partial x_1^2} + \cdots + \frac{\partial^2\phi}{\partial x_N^2}$ is the Laplacian of ϕ in $\mathbf{R}^N$. On the non-vacuum states ($\rho > 0$), $\vec{u} = \frac{\vec{m}}{\rho}$ is the velocity of the flows.

This system describes the dynamic behavior of many important physical flows including the propagation of electrons in submicron semiconductor devices (cf. [4, 20, 24, 37]) and the biological transport of ions for channel proteins (cf. [5] and the references cited therein). In the hydrodynamic model for semiconductor devices, ρ, $\vec{m}$, and ϕ are the electron density, the current mass, and the electrostatic potential,