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Quantization of Poisson–Lie Groups and Applications

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Abstract: Let G be a connected Poisson–Lie group. We discuss aspects of the question of Drinfel'd: *can G be quantized*? and give some answers. When G is semisimple (a case where the answer is *yes*), we introduce quantizable Poisson subalgebras of $C^\infty(G)$, related to harmonic analysis on G; they are a generalization of F.R.T. models of quantum groups, and provide new examples of quantized Poisson algebras.

Introduction*

Quantization in the framework of deformation theory (deformation quantization [26]) was initiated in [2]. The deformation-quantization program of symplectic structures ([2]) leads to various and deep applications in physics and mathematics (e.g. index theory [8 and 26]). The existence of quantizations was first proved using some technical assumptions (essentially vanishing of the cohomology group where obstruction sits), and then in full generality: actually, *any symplectic Poisson bracket can be quantized* ([9 and 26]).

In his Berkeley report [10b], Drinfel'd proposed a similar program of quantization in the case of Poisson–Lie group structures. Here, deformation of algebras will not be enough: one has to deform Hopf algebra structures. In order to quantify the standard Poisson–Lie bracket on simple groups, Drinfel'd introduced deformations of enveloping algebras, which became very popular under the name of *quantum groups*. These structures contain a lot of combinatorial information, and happened to have fundamental applications (e.g. knot theory [19a and 23]), via their universal R-matrix. In [10d], a very natural question was asked: *can any Poisson–Lie structure be quantized*? This question is, in fact, a multivalued one, since it can be given at least three interpretations, each of which leads to a particular problem, that we shall now state.

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* Unexplained notations used in this introduction can be found in Sect. 1, Sect 4 (for Poisson algebras, Poisson–Lie groups, etc) and Appendix A (for representations of semisimple Lie groups)