Commun. Math. Phys. 178, 597–626 (1996)

Communications in Mathematical Physics

Determinant Bundles, Manifolds with Boundary and Surgery

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Received: 28 August 1995 / Accepted: 30 October 1995

Abstract: We define determinant bundles associated to the following data: (i) a family of generalized Dirac operators on even dimensional manifolds with boundary, (ii) the choice of a spectral section for the family of Dirac operators induced on the boundary. Under the assumption that the operators of the boundary family have null spaces of constant dimension we define, through the notion of b-zeta function, a Quillen metric. We also introduce the analogue of the Bismut–Freed connection. We prove that the curvature of a natural perturbation of the Bismut–Freed connection equals the 2-form piece in the right-hand side of the family index formula, thus extending to manifolds with boundary results of Quillen, Bismut and Freed. Given a closed fibration, we investigate the behaviour of the Quillen metric and of the Bismut–Freed connection under the operation of surgery along a fibering hypersurface. We prove, in particular, additivity formulae for the curvature and for the logarithm of the holonomy.

Introduction

Determinants of elliptic operators arise frequently in Quantum Field Theory. In the evaluation of path integrals over families of Dirac operators $\eth = (\eth_z)$, $z \in B$, it is desirable to have a *determinant function*, DET: $B \longrightarrow \mathbb{C}$, assigning a complex number to each operator $\eth_z$.

As explained in the fundamental work of Quillen [23] in general it is only possible to assign a determinant line bundle $\det(\eth)$ and a natural section $\sigma \in \mathscr{C}^\infty(B; \det(\eth))$ to any such Dirac family. If the determinant line bundle is trivial then by fixing a trivializing section $\tau \in \mathscr{C}^\infty(B; \det(\eth))$ we can define a determinant function $\mathrm{DET}_\tau \in \mathscr{C}^\infty(B)$ by comparing the two sections $\sigma, \tau : \sigma(z) = \mathrm{DET}_\tau(z)\tau(z)$. In the physics literature the obstruction to find such a trivializing section is referred to as an anomaly. Even if the determinant line bundle is trivial there are many possible choices of τ and it is natural to try to determine a canonical one.

In the case of $\overline{\partial}$-operators acting on a vector bundle E over a Riemann surface M, thus with parameter space B equal to the space $\mathscr{A}$ of holomorphic structures