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Topologically Nontrivial Field Configurations in Noncommutative Geometry

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Abstract: In the framework of noncommutative geometry we describe spinor fields with nonvanishing winding number on a truncated (fuzzy) sphere. The corresponding field theory actions conserve all basic symmetries of the standard commutative version (space isometries and global chiral symmetry), but due to the noncommutativity of the space the fields are regularized and they contain only a finite number of modes.

1. Introduction

The basic notions of the noncommutative geometry were developed in [2, 3], and, in the form of the matrix geometry, in [4, 5]. The essence of this approach consists in reformulating first the geometry in terms of commutative algebras and modules of smooth functions, and then generalizing them to their noncommutative analogues.

In standard field theory, to any point x of some space(-time) manifold M the values of various fields are assigned:

$$x \in M \longrightarrow \Phi(x), A(x), \dots ,$$

as sections of some bundles over M, e.g. the line bundle of functions, or the spinor bundle, etc. The smooth functions on M form a commutative algebra $\mathscr{A} = \mathscr{F}(M)$ with respect to the standard pointwise product: $(fg)(x) = f(x)g(x)$, $x \in M$. The bundle of smooth spinor fields $\mathscr{S}(M)$ on M is an $\mathscr{A}$-module with respect to the multiplication by smooth functions, which simply means that any spinor can be multiplied by a scalar field. In the same way the linear spaces of gauge and other fields are $\mathscr{A}$-modules. If there exists a sequence of deformations of the commutative algebra $\mathscr{A}$ of the smooth functions on the manifold M, such that the deformed algebras are finite dimensional, we may attempt to formulate a deformed field theory which would possess just a finite number of degrees of freedom. Needless to say,

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