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The Robinson–Trautman Type III Prolongation Structure Contains K_2

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Abstract: The minimal prolongation structure for the Robinson–Trautman equations of Petrov type III is shown to always include the infinite-dimensional, contragredient algebra, K_2, which is of infinite growth. Knowledge of faithful representations of this algebra would allow the determination of Bäcklund transformations to evolve new solutions.

The general class of Robinson–Trautman solutions [1] to the vacuum Einstein field equations have been important examples of exact solutions for many years, albeit they seem to have various difficulties with respect to their interpretation [2]. They are solutions characterized by having a repeated principal null direction, which is of course geodesic and shearfree, and is required to be diverging but not twisting. The standard reference [3] gives the general form of the metric which any Einstein space must have if it permits such a repeated principal null direction, and notes that all possible algebraically-special Petrov types are allowed. In the case of Petrov type III, the field equations are [3] first reduced to

$$K = \Delta \log P \equiv 2P^2 \partial_\zeta \partial_{\overline{\zeta}} \log P = -3[f(\zeta, u) + \overline{f}(\overline{\zeta}, u)] , \tag{1}$$

where K is the Gaussian curvature of the 2-surface spanned by ζ and $\overline{\zeta}$. This equation determines the general RT-solution of Petrov type III. However, since u is nowhere explicitly mentioned within the partial differential equation (pde), it is well-known [3] that one could always simply ignore that dependence, perform a coordinate transformation sending $f(\zeta) \to \zeta$, leaving the curvature completely invariant, and reducing our equation to the rather simple-appearing equation

$$K = 2P^2 \partial_\zeta \partial_{\overline{\zeta}} \log P = -\frac{3}{2}(\zeta + \overline{\zeta}) ,$$

or

$$u_{xy} = \frac{1}{2}(x + y)e^{-2u}, \quad \text{where } \log P \equiv u , \tag{2}$$