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Generalized Variational Principles, Global Weak Solutions and Behavior with Random Initial Data for Systems of Conservation Laws Arising in Adhesion Particle Dynamics

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Abstract: We study systems of conservation laws arising in two models of adhesion particle dynamics. The first is the system of free particles which stick under collision. The second is a system of gravitationally interacting particles which also stick under collision. In both cases, mass and momentum are conserved at the collisions, so the dynamics is described by 2×2 systems of conservations laws. We show that for these systems, global weak solutions can be constructed explicitly using the initial data by a procedure analogous to the Lax–Oleinik variational principle for scalar conservation laws. However, this weak solution is not unique among weak solutions satisfying the standard entropy condition. We also study a modified gravitational model in which, instead of momentum, some other weighted velocity is conserved at collisions. For this model, we prove both existence and uniqueness of global weak solutions. We then study the qualitative behavior of the solutions with random initial data. We show that for continuous but nowhere differentiable random initial velocities, all masses immediately concentrate on points even though they were continuously distributed initially, and the set of shock locations is dense.

1. Introduction

This paper has two main goals: The first is to give an explicit construction of weak solutions for the initial value problem of the systems of conservation laws:

$$\begin{cases} \rho_t + (\rho u)_x = 0 \\ (\rho u)_t + (\rho u^2)_x = 0 \end{cases} \tag{1.1}$$

and

$$\begin{cases} \rho_t + (\rho u)_x = 0 \\ (\rho u)_t + (\rho u^2)_x = -\rho g_x \\ g_{xx} = \rho \,. \end{cases} \tag{1.2}$$

The second is to study the qualitative behavior of such weak solutions when initial data are random. We prove that for a wide class of probability distributions for the