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Floquet Hamiltonians with Pure Point Spectrum

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Abstract: We consider Floquet Hamiltonians of the type $K_F := -i\partial_t + H_0 + \beta V(\omega t)$, where H_0, a selfadjoint operator acting in a Hilbert space $\mathscr{H}$, has simple discrete spectrum $E_1 < E_2 < \cdots$ obeying a gap condition of the type $\inf\{n^{-\alpha}(E_{n+1} - E_n); n = 1, 2, \ldots\} > 0$ for a given $\alpha > 0$, $t \mapsto V(t)$ is 2π-periodic and r times strongly continuously differentiable as a bounded operator on $\mathscr{H}$, ω and β are real parameters and the periodic boundary condition is imposed in time. We show, roughly, that provided r is large enough, β small enough and ω non-resonant, then the spectrum of K_F is pure point. The method we use relies on a successive application of the adiabatic treatment due to Howland and the KAM-type iteration settled by Bellissard and extended by Combescure. Both tools are revisited, adjusted and at some points slightly simplified.

1. Introduction

Spectral analysis of Floquet Hamiltonians or, equivalently, Floquet operators [7, 17] is known to be a tool to investigate the dynamical stability of a quantum system at least in the spirit of the RAGE theorem (see e.g. [6]). If $K = -i\partial_t + H(t)$ is pure point, then for all initial conditions ψ_0 in $\mathscr{H}$, the solution ψ_t of the Schrödinger equation fulfills $\lim_{a\to\infty} \sup_t \|\mathscr{E}(|A| > a)\psi_t\| = 0$, with $\mathscr{E}$ being the spectral measure of an arbitrary self-adjoint operator A. Though it has been realized recently that such information is rather incomplete; in particular it seems that one cannot predict the time behaviour of $\langle\psi_t, A\psi_t\rangle$ from the nature of $\sigma(K)$ (see e.g. [3]).

This paper is concerned with Floquet Hamiltonians $K_F := -i\partial_t + H_0 + \beta V(\omega t)$ depending on two real parameters β and ω. The unperturbed (true) Hamiltonian H_0 in a Hilbert space $\mathscr{H}$ has a simple discrete spectrum $\sigma(H_0) = \{E_1, E_2, \ldots\}$ obeying the gap condition given in (2.1) below (with $\alpha > 0$). The family $V(t)$ is 2π-periodic and sufficiently many times strongly differentiable.

Except for the methods based on randomizing of some parameter [8, 4] and using the Kotani's trick [13, 16], two approaches are known to analyze the spectrum of K_F. The first one is called here the KAM-type iteration method and it was introduced and popularized by Bellissard [1]. This method requires some kind