

On the Universal R -Matrix of $U_q\widehat{sl}_2$ at Roots of Unity

T. Hakobyan, A. Sedrakyan

Service de Physique Théorique de Saclay Laboratoire de la Direction des Sciences de la Matière du Commissariat à l'Energie Atomique, CE-Saclay, F-91191 Gif-sur-Yvette, France. Permanent address: Yerevan Physics Institute, Yerevan, Armenia. E-mail: sedrak@erphy.armenia.su

Abstract: We show that the action of the universal R -matrix of the affine $U_q\widehat{sl}_2$ quantum algebra, when q is a root of unity, can be renormalized by some scalar factor to give a well-defined nonsingular expression, satisfying the Yang–Baxter equation. It can be reduced to intertwining operators of representations, corresponding to Chiral Potts, if the parameters of these representations lie on the well-known algebraic curve.

We also show that the affine $U_q\widehat{sl}_2$ for q is a root of unity from the autoquasi-triangular Hopf algebra in the sense of Reshetikhin.

1. Introduction

The intertwining operators of quantum groups ([1–4]) lead to solutions of the Yang–Baxter equation, which play the crucial role in two dimensional field theory and integrable statistical systems ([4, 5]). It is well-known that most of them can be obtained from the universal R -matrix ([1]) for a given quantum group: the solutions of the spectral parameter dependent Yang–Baxter equation can be obtained from the universal R -matrix of affine quantum groups ([6]) and the solutions of the non-spectral parameter dependent Yang–Baxter equations can be obtained from the universal R -matrix of finite quantum groups.

The situation is not the same for the case when the parameter q of the quantum group is a root of unity.

In this case the center of the quantum group is larger and a new type of representations appear, which have no classical analog ([5, 7, 8, 9]). It was shown in [10, 11] that the cyclic representations lead to solutions of the Yang–Baxter equation with a spectral parameter, lying on some algebraic curve. These solutions correspond to Chiral Potts Model ([12–14]) and its generalizations (for quantum groups $U_q\widehat{sl}_n$).

The formal expression of the universal R -matrix fails in this case: it has singularities when q is a root of unity. Recently in [15] Reshetikhin introduced the notion