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Scaling Limit for a Mechanical System of Interacting Particles

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Abstract: A system of a large number of classical particles moving on a one-dimensional segment with virtually reflecting boundaries is studied. The particles interact with one another through repulsive pair-potential forces and are subjected to resistance proportional to their velocities. Because of the latter it is only the number of particles that is conserved under the evolution of the system. It is proved that in the hydrodynamic limit of diffusion type scaling the normalized counting measure of particle locations converges and its limiting density is governed by a non-linear diffusion equation which in typical cases is of porous media equation type.

0. Introduction

This paper concerns the problem of a hydrodynamic limit for a system of a large number of particles that move on a one-dimensional segment according to a classical equation of motion. Particles interact by exerting upon each other repulsive potential forces given by a common pair-potential function that is unbounded at zero and may have an infinite range of influence. Also at both ends of the segment there act potential forces that repel particles approaching one of the endpoints and keep them in between. In addition the medium exerts on each particle a damping force, "resistance," whose magnitude is proportional to the velocity of the particle, so that nothing except the number of particles is conserved under the evolution of the system. We take the hydrodynamic scaling limit for the normalized counting measure of particle locations in the system and prove that its limiting density is governed by a non-linear diffusion equation that in typical cases is of porous media equation type.

An approach that has been recently developed for studying the hydrodynamic limits of stochastic systems is applied to the present model. Although our system is deterministic, the application goes well owing to the dissipative effect of resistance and the fact that our system is essentially of gradient type. The idea for the general