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# The Momentum Constraints of General Relativity and Spatial Conformal Isometries

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**Abstract:** Transverse-tracefree (TT-) tensors on $(\mathbf{R}^3, g_{ab})$, with $g_{ab}$ an asymptotically flat metric of fast decay at infinity, are studied. When the source tensor from which these TT tensors are constructed has fast fall-off at infinity, TT tensors allow a multipole-type expansion. When $g_{ab}$ has no conformal Killing vectors (CKV's) it is proven that any finite but otherwise arbitrary set of moments can be realized by a suitable TT tensor. When CKV's exist there are obstructions – certain (combinations of) moments have to vanish – which we study.

## 1. Introduction

In this paper we consider transverse-tracefree (TT-) tensors on $\mathbf{R}^3$ with an asymptotically flat metric $g_{ab}$, i.e. tensors $P_{ab}$ satisfying

$$D^a P_{ab} = 0, \quad \text{trace } P = 0 \text{ on } (\mathbf{R}^3, g_{ab}), \tag{1.1}$$

where $D$ is the covariant derivative associated with $g$. The interest in this problem comes first of all from (vacuum) general relativity, where Eq. (1.1) is the momentum constraint for an initial data set $(\mathbf{R}^3, g_{ab}, P_{ab})$

$$D^a(P_{ab} - g_{ab} \text{ trace } P) = 0 \tag{1.2}$$

in the maximal (i.e. trace $P = 0$) case. As is well-known, Eq. (1.2) is just the expression of the invariance of the theory under diffeomorphisms of three space. Thus our study of Eq. (1.1) is relevant to a much larger class of theories than Einstein's.

In the standard conformal approach to solving the constraints [13], Eq. (1.1) is not solved on the physical metric $g_{ab}$, but a conformally related metric $g'_{ab}$ having faster decay at infinity than $g_{ab}$. One is here using the fact that $P_{ab}$ being $TT$ is

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