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Viscosity for a Periodic Two Disk Fluid: An Existence Proof

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Summary. We express the momentum current (= stress) tensor for a periodic fluid with two hard disks per unit cell in terms of a single particle billiard. We establish a central limit theorem for the time-integrated stress tensor and thereby prove the existence of a strictly positive shear and bulk viscosity.

1. Introduction

One of the great challenges of statistical mechanics is to prove the existence of finite (and non-zero) transport coefficients for a system of particles governed by Newton's equations of motion. For a one component fluid these transport coefficients are the shear and bulk viscosity and the thermal conductivity. There are several, presumably equivalent, ways to define them – the clearest and least ambiguous of which is through the Green-Kubo formula. Let us briefly recall the basic structure. We consider an infinitely extended, one component fluid in thermal equilibrium. The equilibrium average is denoted by $\langle \cdot \rangle$. In three dimensional physical space the fluid has five locally conserved fields: the particle density $n^{(0)}(x,t)$, the three components of the momentum density $n^{(\alpha)}(x,t)$, $\alpha = 1,2,3$, and the energy density $n^{(4)}(x,t)$, which depend on location $x \in \mathbb{R}^3$ and time $t \in \mathbb{R}$. [These are distributions on phase space indexed by x, t. Their precise form is of no importance for what follows. More details can be found in [17,21].] By the local conservation law we have, in a distributional sense,

$$\frac{\partial}{\partial t} n^{(i)}(x,t) + \operatorname{div} j^{(i)}(x,t) = 0, \tag{1.1}$$

$i = 0, \ldots, 4$, with the local currents $j^{(i)}$. [Since the interaction between particles has some range, the local currents are not uniquely defined. However, the space averaged currents always are, cf. Sect. 2.] The Green-Kubo formula for the transport coefficients reads then