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Neumann Resonances in Linear Elasticity for an Arbitrary Body

P. Stefanov[1,★,★★], **G. Vodev**[2]

[1] Institute of Mathematics, Bulgarian Academy of Sciences, Acad. G. Bontchev str., bl. 8, 1113 Sofia, Bulgaria

[2] Université de Rennes 1, IRMAR, URA 305 du CNRS, Campus de Beaulieu, F-35042 Rennes Cedex, France

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Abstract: We study resonances (scattering poles) associated to the elasticity operator in the exterior of an arbitrary obstacle in $\mathbf{R}^3$ with Neumann boundary conditions. We prove that there exists a sequence of resonances tending rapidly to the real axis.

1. Introduction

Let $\mathcal{O} \subset \mathbf{R}^3$ be a compact set with C^∞-smooth boundary Γ and connected complement $\Omega = \mathbf{R}^3 \backslash \mathcal{O}$. Denote by Δ_e the elasticity operator

$$\Delta_e v = \mu_0 \Delta v + (\lambda_0 + \mu_0) \nabla (\nabla \cdot v),$$

$v = {}^t(v_1, v_2, v_3)$. Here λ_0, μ_0 are the Lamé constants and we assume that

$$\mu_0 > 0, \qquad 3\lambda_0 + 2\mu_0 > 0. \tag{1}$$

Consider Δ_e in Ω with Neumann boundary conditions on Γ,

$$\left. \sum_{j=1}^{3} \sigma_{ij}(v) \nu_j \right|_{\Gamma} = 0, \quad i = 1, 2, 3, \tag{2}$$

where $\sigma_{ij}(v) = \lambda_0 \nabla \cdot v \delta_{ij} + \mu_0 \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$ is the stress tensor, ν is the outer normal to Γ. Denote by L the self-adjoint realization of $-\Delta_e$ in Ω with Neumann boundary conditions on Γ. As usual we define *resonances* as the poles of the meromorphic continuation of the cut-off resolvent $R_\chi(\lambda) = \chi(L - \lambda^2)^{-1}\chi$ from $\operatorname{Im} \lambda < 0$ to the whole complex plane $\mathbf{C}$, $\chi \in C_0^\infty$ being a cut-off function equal to 1 near Γ. So we accept the convention that the resonances lie in the upper half-plane.

If one considers the Laplacian with Dirichlet or Neumann boundary conditions, then it is well known that for convex or more generally for non-trapping obstacles

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★★ Current address: Department of Mathematics, University of British Columbia, Vancouver, B.C., V6T 1Y4, Canada.