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Gelfand-Tzetlin Coordinates for the Unitary Supergroup

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Abstract: The Gelfand-Tzetlin method provides explicit coordinates on the parameter space of the unitary group $U(k)$ which make direct evaluations of group integrals possible. It is closely related to the Gelfand construction of finite-dimensional irreducible representations. We generalize the Gelfand-Tzetlin method to the unitary supergroup $U(k_1/k_2)$. The coordinates on the parameter space for supergroup integrals and the invariant Haar measure are evaluated. As an example, the supersymmetric Harish-Chandra-Itzykson-Zuber integral is calculated. A generalized Gelfand pattern containing anticommuting variables is introduced which determines the representation.

1. Introduction

Almost half a century ago, Gelfand [6] constructed representations of the unitary group based on a chain of unitary subgroups in which the dimension is lowered by one in every step. The advantage of using such a recursive embedding becomes obvious in the structure of the Gelfand patterns which determine these representations and results naturally in many applications in physics [4, 2]. At the same time, Gelfand and Tzetlin [8, 7] constructed explicit coordinates on the parameter space of the unitary group which are the interpretation of the discrete integers in the Gelfand pattern as continuous variables. Hence, this so-called Gelfand-Tzetlin method is closely related to representation theory. Recently, Shatashvili [17] used the Gelfand-Tzetlin coordinates in order to evaluate the correlation functions in the Itzykson-Zuber model [15, 14]. These calculations reflect the recursive structure of the Gelfand-Tzetlin method directly.

The purpose of this paper is the generalization of the Gelfand-Tzetlin method to the unitary supergroup. Berezin [3] found a nontrivial extension of the standard analysis by employing anticommuting degrees of freedom. He also extended the standard algebra and group theory by defining supermatrices and supergroups. After the pioneering work of Efetov [5], these tools are being used in many areas of physics [18]. Furthermore, a supersymmetric generalization of the Harish-Chandra-Itzykson-Zuber

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