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Conformal Haag–Kastler Nets, Pointlike Localized Fields and the Existence of Operator Product Expansions

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Abstract: Starting from a chiral conformal Haag–Kastler net on 2 dimensional Minkowski space we construct associated pointlike localized fields. This amounts to a proof of the existence of operator product expansions.

We derive the result in two ways. One is based on the geometrical identification of the modular structure, the other depends on a "conformal cluster theorem" of the conformal two-point-functions in algebraic quantum field theory.

The existence of the fields then implies important structural properties of the theory, as PCT-invariance, the Bisognano–Wichmann identification of modular operators, Haag duality and additivity.

1. Introduction

The formulation of quantum field theory in terms of Haag Kastler nets of local observable algebras ("local quantum physics" [Haag]) has turned out to be well suited for the investigation of general structures. Discussion of concrete models, however, is mostly done in terms of pointlike localized fields.

In order to be in a precise mathematical framework, these fields might be assumed to obey the Wightman axioms [StW]. Even then, the interrelation between both concepts is not yet completely understood (see [BaW, BoY] for the present stage).

In the Wightman framework, the postulated existence of operator product expansions [Wi1] has turned out to be very fruitful, especially in $2d$ conformal field theory. The existence of a convergent expansion of the product of two fields on the vacuum could be derived from conformal covariance, but the existence of the associated local fields had to be postulated [Lüs, Mac, SSV].

In the Haag–Kastler framework, the existence of an operator product expansion might be formulated as the existence of sufficiently many Wightman fields such that their linear span applied to the vacuum is dense in the Hilbert space. Actually, we

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