Commun. Math. Phys. 176, 479 – 539 (1996)

Communications in
Mathematical
Physics

Classification of Local Generalized Symmetries for the Vacuum Einstein Equations

Ian M. Anderson[1], **Charles G. Torre**[2]

[1] Department of Mathematics, Utah State University, Logan, UT 84322-3900 USA
[2] Department of Physics, Utah State University, Logan, UT 84322-4415 USA

Received: 21 April 1994 / in revised form: 19 June 1995

Dedicated to the memory of H. Rund

Abstract: A local generalized symmetry of a system of differential equations is an infinitesimal transformation depending locally upon the fields and their derivatives which carries solutions to solutions. We classify all local generalized symmetries of the vacuum Einstein equations in four spacetime dimensions. To begin, we analyze symmetries that can be built from the metric, curvature, and covariant derivatives of the curvature to any order; these are called natural symmetries and are globally defined on any spacetime manifold. We next classify first-order generalized symmetries, that is, symmetries that depend on the metric and its first derivatives. Finally, using results from the classification of natural symmetries, we reduce the classification of all higher-order generalized symmetries to the first-order case. In each case we find that the local generalized symmetries are infinitesimal generalized diffeomorphisms and constant metric scalings. There are no non-trivial conservation laws associated with these symmetries. A novel feature of our analysis is the use of a fundamental set of spinorial coordinates on the infinite jet space of Ricci-flat metrics, which are derived from Penrose's "exact set of fields" for the vacuum equations.

1. Introduction

Symmetry plays an important role throughout theoretical physics and one of central importance in field theory [1, 2]. Indeed, in the construction of a field theory physical considerations usually demand that the field equations (or the Lagrangian) possess certain symmetries. These symmetries include Poincaré symmetry, gauge symmetry, diffeomorphism symmetry, various discrete symmetries, and a host of specialized symmetries needed to ensure the conservation of appropriate quantum numbers. Symmetries also play an important role in the mathematical analysis of differential equations [3, 4]. Originating with the work of Lie, symmetry group methods and their recent generalizations have proved useful in understanding conservation laws,