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Quasi-Galois Symmetries of the Modular S-Matrix

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Abstract: The recently introduced Galois symmetries of rational conformal field theory are generalized, for the case of WZW theories, to "quasi-Galois symmetries." These symmetries can be used to derive a large number of equalities and sum rules for entries of the modular matrix S, including some that previously had been observed empirically. In addition, quasi-Galois symmetries allow us to construct modular invariants and to relate S-matrices as well as modular invariants at different levels. They also lead us to a convenient closed expression for the branching rules of the conformal embeddings $\mathbf{g} \hookrightarrow \widehat{\mathrm{so}}(\dim \bar{\mathbf{g}})$.

1. Introduction

In the study of rational conformal field theories, modular transformations play an essential role. They turn the set of the characters of all primary fields into a unitary module of $SL(2, \mathbb{Z})$, the twofold covering of the modular group of the torus. Via the Verlinde formula, they are also closely related to the fusion rules.

In all cases where the modular matrix S is explicitly known, one observes that it contains surprisingly few different numbers, and that among the distinct numbers there are linear relations. While it has been known for a long time that simple currents lead to relations between individual S-matrix elements [1–3], many other relations, in particular sum rules, have remained so far somewhat mysterious. Recently it has become clear that Galois symmetries [4, 5] are an independent source for relations between individual elements of S [6, 7]. Both simple current and Galois symmetries exist for arbitrary rational conformal field theories, independent of the structure of the chiral algebra.

In this paper we will show that in the special case of WZW theories, Galois symmetries can be generalized to what we will call *quasi-Galois symmetries*. A crucial ingredient of our construction (which is not available for other conformal field theories than WZW theories) is the Kac–Peterson formula for the S-matrix. These new symmetries turn out to be rather powerful and allow us to derive three

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