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Non-linear Structures in the Non-critical NSR String

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Abstract: We investigate the Ward identities of the $\mathcal{W}_\infty$ symmetry in the super-Liouville theory coupled to the super-conformal matter of central charge $\hat{c}_M = 1 - 2(p-q)^2/pq$. The theory is classified into two chiralities. For the positive chirality, all gravitationally dressed scaling operators are generated from the $q-1$ gravitational primaries by one of the ring generators in the R-sector acting on them repeatedly. After fixing the normalizations of the dressed scaling operators, we find that the Ward identities are expressed in the form of the *usual* $\mathcal{W}_q$ algebra constraints as in the bosonic case: $\mathcal{W}_n^{(k+1)}\tau = 0$, $(k = 1, \ldots, q-1;\ n \in \mathbf{Z}_{\geqq 1-k})$, where the equations for even and odd n come from the currents in the NS- and the R-sector respectively. The non-linear terms come from the anomalous contributions at the boundaries of moduli space. The negative chirality is defined by interchanging the roles of p and q. Then we get the $\mathcal{W}_p$ algebra constraints.

1. Introduction

One of the prominent features of the non-critical string, or 2D quantum gravity is the appearance of the non-linear structures [1–6], which have been first derived in the double scaling limit of the matrix model [1] and then developed in the form of the $\mathcal{W}$-algebra constraints [3, 4]. Recently one of the authors (K.H.) [6] has found that the $\mathcal{W}$-algebra constraints are realized as the Ward identities of $\mathcal{W}_\infty$ symmetry [7] in the Liouville theory approach [5–13]. The scaling operators are then identified with the tachyon-like operators with discrete momenta. The non-linear terms come from the anomalous contributions from the boundaries of moduli space.

In this paper we investigate the non-critical NSR string, or 2D quantum supergravity. In this case the application of the continuum approach is extremely relevant because there is no definite matrix model to describe it. The BRST analysis of the physical spectrum was recently carried out [14]. It was then found that the ring structure and the associated $\mathcal{W}_\infty$ symmetry algebra are the same as those in the

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