Commun. Math. Phys. 176, 321 – 351 (1996)

Communications in
Mathematical
Physics


# Infinite Dimensional Groups and Riemann Surface Field Theories

**A.L. Carey[1], K.C. Hannabuss[2]**

[1] Department of Pure Mathematics, University of Adelaide, Adelaide 5005, Australia
[2] Balliol College, Oxford, U.K.

Received: 4 February 1994 / in revised form: 10 April 1995

**Abstract:** We show how to obtain positive energy representations of the group $\mathcal{G}$ of smooth maps from a union of circles to $U(N)$ from geometric data associated with a Riemann surface having these circles as boundary. Using covering spaces we can reduce to the case where $N = 1$. Then our main result shows that Mackey induction may be applied and yields representations of the connected component of the identity of $\mathcal{G}$ which have the form of a Fock representation of an infinite dimensional Heisenberg group tensored with a finite dimensional representation of a subgroup isomorphic to the first cohomology group of the surface obtained by capping the boundary circles with discs. We give geometric sufficient conditions for the correlation functions to be positive definite and derive explicit formulae for them and for the vacuum (or cyclic) vector. (This gives a geometric construction of correlation functions which had been obtained earlier using tau functions.) By choosing particular functions in $\mathcal{G}$ with non-zero winding numbers on the boundary we obtain analogues of vertex operators described by Segal in the genus zero case. These special elements of $\mathcal{G}$ (which have a simple interpretation in terms of function theory on the Riemann surface) approximate fermion (or Clifford algebra) operators. They enable a rigorous derivation of a form of boson-fermion correspondence in the sense that we construct generators of a Clifford algebra from the unitaries representing these elements of $\mathcal{G}$.

## Introduction

The aim of this paper is to show how geometric data associated to a Riemann surface lead naturally to unitary representations of infinite dimensional Lie groups and representations of Clifford and Heisenberg algebras. Our study is related to an extensive earlier literature. However, we have attempted to make our discussion comparatively self contained.

Initially we were motivated by a desire to understand some of the literature on conformal quantum field theory [A-GMV, A-GNMV, A-GBNMV, E]. Significant progress in this direction has come from the algebraic approach of [DJKM, KNTY]. The starting point of the latter is the so-called tau function and its relation to soliton