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Dirichlet Form Approach to Infinite-Dimensional Wiener Processes with Singular Interactions

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Dedicated to Professor Masatoshi Fukushima on his 60th birthday

Abstract: We construct infinite-dimensional Wiener processes with interactions by constructing specific quasi-regular Dirichlet forms. Our assumptions are very mild; accordingly, our results can be applied to singular interactions such as hard core potentials, Lennard–Jones type potentials, and Dyson's model. We construct non-equilibrium dynamics.

0. Introduction

Infinite-dimensional Wiener processes with interactions are diffusion processes with state space $(\mathbb{R}^d)^{\mathbb{N}}$ (or Θ, where Θ is the set of all locally finite configurations of particles on $\mathbb{R}^d$) with interactions. When interactions come from a smooth pair potential Φ and martingale terms have constant coefficients 1, these processes are described by the following SDE;

$$dX_t^i = dB_t^i - \sum_{j=1, j\neq i}^{\infty} \frac{1}{2}\nabla\Phi(X_t^i - X_t^j)\,dt \quad (1 \leqq i < \infty), \tag{0.1}$$

where B_t^i $(1 \leqq i < \infty)$ are independent Brownian motion on $\mathbb{R}^d$, and $\Phi : \mathbb{R}^d \to \mathbb{R}$. The associated Θ-valued process is

$$\mathbb{X}_t = \sum_{i=1}^{\infty} \delta_{X_t^i} \quad (\delta_a \text{ is the delta measure at } a.) \tag{0.2}$$

The study of (0.1) has been initiated by Lang [La1, 2]. He solved (0.1) under suitable conditions on interactions for a set of initial configurations. Shiga [Sh] completed a gap of Lang's proof. Initial configurations for which (0.1) is solved were specified by Lippner [Li] and Rost [Ro] for $d = 1$, and Fritz [F] for $d \leqq 4$.

Since Lang used SDE approach, a smoothness of Φ was crucial. He assumed:

(L.1) $\Phi \in C_0^3(\mathbb{R}^d)$, that is, Φ is finite range and of class C^3.
(L.2) Φ is super stable in the sense of Ruelle.

As a consequence some interesting examples were excluded.